PHYSICAL REVIEW E VOLUME 55, NUMBER 5 MAY 1997

Parametric correlations of phase shifts and statistics of time delays in quantum chaotic scattering: Crossover between unitary and orthogonal symmetries

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(Received 15 January 1997)

We analyze universal statistical properties of phase shifts and time delays for open chaotic systems in the crossover regime of partly broken time-reversal invariance. In particular, we find that the distribution of the time delay shows $\tau^{-3/2}$ behavior for weakly open systems of any symmetry. [S1063-651X(97)51205-0]

PACS number(s): 05.45.+b, 24.30.−v

The energy-dependent scattering phase shifts $\theta_a(E)$ defined via the eigenvalues $\exp i\theta_a$, $a=1,\ldots,M$ of the $M\times M$ unitary scattering matrix $\hat{S}(E)$ are important and frequently used characteristics of the process of quantum scattering. In particular, the derivatives of phase shifts over energy $\tau_a=\partial\theta_a/\partial E$ are related to the duration of a collision event. For example, the quantity $\tau_W=M^{-1}\Sigma_a\tau_a$ is the typical time delay due to scattering, the so-called Wigner-Smith time delay [1]. When some external parameters are taken into consideration (e.g., a magnetic field) the corresponding parametric variation of the phase shifts can as well be related to some observables [2].

Growing interest in the universal features [3] of quantum systems, whose classical counterparts demonstrate chaotic dynamics, attracted considerable attention to the process of quantum chaotic scattering, see [4] and [5] and references therein. From this point of view, different statistical characteristics of phase shifts and time delays were addressed in experiments on chaotic microwave reflection [6], as well as in several numerical studies of various models of quantum scattering in disordered and chaotic systems [7–10].

It is interesting to mention that for the case of only two open channels $M=2$ the phase shifts $\theta_{1,2}$ can simply be related to the phases of transmission and reflection coefficients, see, e.g., [11]. The latter quantities are amenable to direct experimental measurements in quantum dots, see [12] and references therein. Another fact attributing additional interest to studies of time delay statistics is that it is intimately connected with the issue of mesoscopic fluctuations of dynamic admittances of microstructures as shown by Gopar, Mello, and Büttiker [13].

One can extract statistical characteristics of the S matrix exploiting a semiclassical periodic orbit expansion like that provided by the Gutzwiller trace formula, see examples of such calculations in [4–6]. The resulting expressions provide an important insight into the problem. In particular, the semiclassical approximation for the time delay correlations at two different energies was derived by Eckhardt [14]. However, the results obtained in such a way have a restricted domain of applicability; in particular, they fail to describe the system with only a few open channels: $M\sim 1$.

A powerful alternative to the semiclassical methods in extracting the *universal* (i.e., generic and system independent) statistical characteristics of the scattering matrix is provided by the random matrix approach. In particular, in the "Heidelberg variant" of this approach [15] one relates the scattering matrix $S(E)$ to the Hamiltonian of a closed counterpart of the open system. The latter Hamiltonian is considered to be a member of an ensemble of random matrices of appropriate global symmetry—an idea commonly accepted in the domain of quantum chaos. The use of random matrices gives one the possibility to apply the very powerful machinery of averaging developed by Efetov [16] and to calculate different correlation functions explicitly for any number of open channels and arbitrary coupling to continua.

When employing the Heidelberg method the actual calculation depends quite essentially on the symmetry of the underlying Hamiltonian. The simplest case to study corresponds to completely broken time-reversal invariance (TRI) (systems in a strong enough magnetic field), when the random matrix Hamiltonian $\hat{H}$ is taken from the Gaussian unitary ensemble (GUE). For such systems, the statistics of phase shifts, delay times, and resonance poles was thoroughly investigated recently by two of us [17], see a detailed exposition of the calculation in [18]. In the opposite case of fully preserved TRI when $\hat{H}$ is a member of the Gaussian orthogonal ensemble (GOE) some aspects of time evolution of a chaotic system were considered in [20], the correlation function of Wigner-Smith time delays for two different energies was found in [19], and the distribution of time delays was obtained for the perfect coupling case in [13], see also [21]. Let us also mention the paper [22] addressing the issue of parametric correlations for S-matrix elements.

In the present Rapid Communication we extend the analysis of statistical properties of phase shifts and time delays to the whole crossover region of gradual breaking of the TRI. Different characteristics of chaotic and disordered systems in this crossover regime were under quite an intensive theoretical investigation recently [23–25].

Within the framework of the random matrix theory, Hamiltonians of the *closed* chaotic systems under consideration are conveniently represented as [23,25]: $\hat{H}(y)=\hat{H}_S+i\,(y/\sqrt{N})\hat{H}_A$, where $\hat{H}_S$ is $N\times N$ GOE matrix and $\hat{H}_A$ is a real random antisymmetric matrix of the same dimension. For the sake of generality the symmetric matrix $\hat{H}_S$ is taken in the form [3]: $\hat{H}_S=\hat{H}_S^{(0)}+(x/\sqrt{N})\hat{H}_S^{(1)}$. This form allows one to simulate the influence of such perturbations (e.g., a variation of the strength of scattering potential) which do not break the TRI. All elements of random matrices are independent and normalized in such a way that $\langle \mathrm{Tr}(\hat{H}_{S,A}^{(0,1)})^2\rangle = N$.

In the limit $N\to\infty$ the crossover is driven by the parameter $y\in[0,\infty)$, with $y=\infty$ corresponding to completely broken TRI. Physically the parameter y is proportional to the magnetix flux through the system Φ. One may also notice that the typical shift of the levels due to the antisymmetric perturbation is $\delta E_y/\Delta \sim y^2$ [25], where Δ is the mean level spacing.

Within the framework of the Heidelberg approach [15] the coupling of the chaotic region to the incoming and outgoing waves is described with the help of the $M\times N$ matrix $\hat{W}$ of amplitudes W_{ia}, $a=1,2,\ldots,M$; $i=1,\ldots,N$, which couple the internal motion to M open channels. In what follows we consider the case of arbitrary, but fixed M whereas $N\to\infty$. Without much loss of generality these amplitudes can be chosen in a way ensuring that the average S matrix is diagonal in the channel basis: $\langle S_{ab}\rangle = \delta_{ab}\langle S_{aa}\rangle$. The strength of coupling to continua is convenient to be characterized via the "sticking probabilities" (also called the "transmission coefficients") $T_a = 1-|\langle S_{aa}\rangle|^2$, which are given for the present model by the following expression [15]:

$$T_a^{-1} = \frac{1}{2}\left[1+\frac{\gamma_a+\gamma_a^{-1}}{2\pi\nu(E)}\right]; \quad \gamma_a\delta_{ab} = \pi\sum_i W_{ia}W_{ib}, \tag{1}$$

with $\nu(E)=\pi^{-1}(1-E^2/4)^{1/2}$ being the density of states for the GOE matrices related to the local mean level spacing as $\Delta=(\nu N)^{-1}$. The quantity T_a measures the part of the flux in channel a that spends a substantial part of time in the interaction region [15]. We see that both limits $\gamma_a\to 0$ and $\gamma_a\to\infty$ equally correspond to the weak effective coupling regime $T_a \ll 1$, whereas the strongest coupling (at fixed energy E) corresponds to the value $\gamma_a=1$. The maximal possible coupling corresponding to the upper bound $T_a=1$ is achieved in the present model for an energy interval in the vicinity of the center $E=0$. Below we restrict our attention to this point of spectrum in order to present our final results in the most compact form. Moreover, we consider all channels to be statistically equivalent: $\gamma_a=\gamma$ for $a=1,\ldots,M$. Generalization to arbitrary E and nonequivalent channels can easily be done, see [18].

In earlier works [17] and [18] it was shown that one can study, very effectively, the statistics of phase shifts θ_a considered mod 2π. To this end we find it to be convenient to introduce the auxiliary "phases" ϕ related to the phase shifts θ as $\phi=\arctan\{\gamma^{-1}\tan(\theta/2)\}$ and consider the density $\rho_{E,x,y}(\phi)=M^{-1}\Sigma_a\delta(\phi-\phi_a(E,x,y))$.

The connected part of the correlation function of these densities is our main object of interest. It can be found performing calculations *mutatis mutandis*, similar to that presented in [17,18] and it turns out to be dependent only on the difference $\phi=\phi_1-\phi_2$

$$\begin{aligned} K^{\phi}_{\omega,x,y_1,y_2}(\phi) &= \langle \rho_{E=0,x=0,y_1}(\phi_1)\rho_{E+\Omega,x,y_2}(\phi_2)\rangle_c \\ &= \mathrm{Re}\int_{-1}^{1} d\lambda \int_1^\infty d\lambda_1 \int_1^\infty \frac{d\lambda_2}{\mathcal{R}^2}\mathcal{F}_M(\phi) e^{-x^2/2\,(2\lambda_1^2\lambda_2^2-\lambda_1^2-\lambda_2^2-\lambda^2+1)} e^{-i\omega(\lambda_1\lambda_2-\lambda)+y_1y_2(\lambda_1^2-\lambda_2^2)-1/2\,(y_1^2+y_2^2)(\lambda_1^2+\lambda_2^2-\lambda^2-1)} \\ &\quad \times\{(1-\lambda^2)\cosh\alpha-(\lambda_1^2-\lambda_2^2)\sinh\alpha+\mathcal{R}[(y_1^2+y_2^2)((1-\lambda^2)\cosh\alpha+(\lambda_2^2-\lambda_1^2)\sinh\alpha) \\ &\quad +2y_1y_2(\lambda_1^2+\lambda_2^2+\lambda^2-1)\sinh\alpha]\} \end{aligned} \tag{2}$$

where the "channel factor" is equal to

$$\mathcal{F}_M(\phi) = -\frac{\partial^2}{\partial\phi^2}\left[\frac{(1+i\lambda\,\tan\phi)^2}{1+2i\lambda_1\lambda_2\tan\phi-\tan^2\phi(\lambda_1^2+\lambda_2^2-1)}\right]^{M/2}, \tag{3}$$

and where $\mathcal{R},\alpha$, and ω denote $\mathcal{R}=\lambda_1^2+\lambda_2^2+\lambda^2-2\lambda\lambda_1\lambda_2-1$, $\alpha=y_1y_2(1-\lambda^2)$, and $\omega=\pi\Omega/\Delta$.

The correlation function presented above is a very informative object. First of all, having it at our disposal, it is a relatively easy task to show [18] that the correlation function of Wigner-Smith time delays $K^{\tau}_{\omega,x,y_1,y_2} = \langle\tau_W(E=0,x=0,y_1)\tau_W(E+\Omega,x,y_2)\rangle/\langle\tau_W(E)\rangle^2$ is given in the crossover regime by the same expression Eq. (2), provided one replaces the "channel factor" Eq. (3) by

$$\mathcal{F}_M^{\tau} = (\lambda_1\lambda_2-\lambda)^2\left[\frac{(g+\lambda)^2}{(g+\lambda_1\lambda_2)^2-(\lambda_1^2-1)(\lambda_2^2-1)}\right]^{M/2}, \tag{4}$$

here $g=2/T-1$, with T being the transmission coefficient introduced above.

Secondly, one can extract explicitly the general distribution function of the scaled partial delay times $\mathcal{P}(\tau_s)=\langle 1/M\Sigma_a\delta(\tau_s-\Delta/2\pi\tau_a(y))\rangle$ in the crossover regime

$$\mathcal{P}(\tau_s) = \frac{C_M}{\tau_s^{(M+5)/2}}\int_{-1}^{1} d\lambda \int_1^\infty d\lambda_2 \lambda_2^{(M+3/2)}(\lambda_2^2-1)^{(1-M)/4} \times e^{-2y^2(\lambda_2^2-1)}\mathcal{J}_1(\lambda_2)\mathcal{J}_2(\lambda,\lambda_2), \tag{5}$$

where $C_M=[(2\pi)^{1/2}2^{M/2+1}\Gamma(M/2+1)]^{-1}$ and

$$\mathcal{J}_1(\lambda_2) = \int_0^\pi d\psi\, v(\psi)^{M+1/2} e^{-(\lambda_2^2/\tau_s)v(\psi)} I_{(M-1)/2}\left[\frac{\lambda_2\sqrt{\lambda_2^2-1}}{\tau_s}v(\psi)\right],$$

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$$\mathcal{J}_2(\lambda,\lambda_2)=4y^2[(1-\lambda^2)e^{-\beta}+\lambda_2^2(1-e^{-\beta})]-(1-e^{-\beta}),$$

where $v(\psi)=g-\sqrt{g^2-1}\cos\psi$, $\beta=2y^2(1-\lambda^2)$, and $I_p(z)$ stands for the modified Bessel function.

The distribution Eq. (5) is valid for any number of open channels M and any value of transmission coefficient T and as such is quite complicated. To get a better understanding of its typical features it is reasonable to look separately at two limiting cases of strong (weak) coupling to continua.

For the strong coupling regime $T=1$ (i.e., $g=1$) the ψ integration in Eq. (5) drops out, but the resulting expression is still quite cumbersome. However, one can easily find the long time asymptotics to be of the following form:

$$\mathcal{P}(\tau_s\gg 1)=\begin{cases} U_M(y)\tau_s^{-(2+M)} & y>0 \\ (2\pi)^{1/2}C_M e^{-1/2\tau_s}\tau_s^{-(2+M/2)} & y=0, \end{cases} \tag{6}$$

where $U_M(y)$ is a rather complicated function of the symmetry breaking parameter y.

The second of this expressions holding for unbroken TRI is actually exact for arbitrary τ_s as can be seen performing the limit $y\rightarrow 0$ in the general Eq. (5) at $T=1$. This fact was first conjectured in [21] and derived for the particular case $M=1$ by another method in [13].

Equation (6) demonstrates that the limits $y\rightarrow 0$ and $\tau\rightarrow\infty$ do not commute. To understand this phenomenon better it is instructive to consider the case of ''weakly broken'' TRI, $y\ll 1$, in more detail. A close inspection shows that for such regime there emerges one more relevant time scale $\tau_y\propto y^{-2}\gg 1$, such that for the domain $1\ll\tau_s\ll\tau_y$ the distribution function $\mathcal{P}(\tau_s)$ shows the GOE-like behavior: $\mathcal{P}(\tau_s)\propto\tau_s^{-(M/2+2)}$, whereas at $\tau_s\gg\tau_y$ this behavior changes to the GUE-like: $\mathcal{P}(\tau_s)\propto\tau_y^{M/2}\tau_s^{-(M+2)}$.

We suggest the following transparent physical interpretation of the scale τ_y: this is just the time $\hbar/\delta E_y$ necessary by the Heisenberg uncertainty relation to resolve a typical shift δE_y due to the TRI-breaking perturbation. If the particle dwells in the scattering domain for a time shorter than τ_y it cannot ''feel'' the magnetic field effects and the corresponding asymptotics is GOE-like. However, for large enough times, the particle resolves the effect of the broken TRI however small is the magnetic field. This explains why the most distant asymptotics of the time delay distribution is always GUE-like, provided the magnetic field is not identically zero.

Let us now turn our attention to the opposite limit of an almost ''closed'' chaotic system: $T\ll 1$. Exploiting $g\gg 1$ we find for arbitrary number of open channels and arbitrary y the following universal (up to a coefficient) behavior of delay time distribution:

$$\mathcal{P}(\tau_s)\propto g^{-1/2}\tau_s^{-3/2} \quad \text{when} \quad g^{-1}\ll\tau_s\ll g \tag{7}$$

in the parametrically large region of delay times. The proportionality coefficient in this formula depends on the parameters y and M in a complicated way.

Such a $\tau^{-3/2}$ behavior, holding irrespective of the TRI symmetry, is the most robust feature of the time delay statistics of weakly open chaotic systems. It was first obtained in [18] for the case of broken TRI, but physical arguments show that it is a very general feature simply following from the picture of well-isolated resonances typical for such systems [18]. It also seems to be quite insensitive to the particular details of definitions of time delays and holds for distributions of such slightly different quantities as Wigner-Smith time delay, partial delay times or even ''dwell times.'' Numerical simulations of dwell times in chaotic scattering on a two dimensional cavity in tunneling contact with two waveguides have been performed in [10]. Sampling the distribution of dwell times from the energy-dependent fluctuating pattern obtained in [10] we find a good agreement with the predicted $\tau^{-3/2}$ intermediate asymptotics, see Fig. 1.

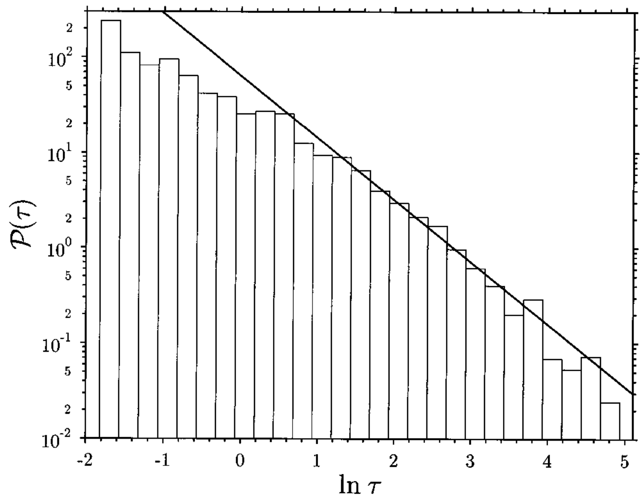

FIG. 1. The distribution of the dwell times τ in chaotic scattering in weakly open Sinai-like billiard [10]. Straight line corresponds to the theoretical prediction $\mathcal{P}(\tau)\propto\tau^{-3/2}$ with fitted proportionality coefficient. Deviations at small (large) τ are due to crossovers to other asymptotical regimes (see text).

Outside the parametrically large interval $g^{-1}\ll\tau_s\ll g$ our general expression Eq. (5) predicts an exponential cutoff at $\tau_s\lesssim g^{-1}$ and a crossover to the behavior described by Eq. (6) for the asymptotically large times $\tau\gg g$. One can check that for ''weakly broken'' TRI again there emerges a scale $\tau_y(g)\sim g/y^2$, such that the asymptotic tail is GOE-like at $g\ll\tau_s\ll\tau_y(g)$, but always GUE-like for $\tau_s\gg\tau_y(g)$, in full agreement with the discussion above.

Finally, having in mind the comparison with the semiclassics let us consider in more detail the large-channel limit $M\gg 1$ of our general expressions like Eq. (2) describing the correlations of phase shifts and Wigner-Smith time delays. When doing this it is natural to consider the angle difference $\phi=\phi_1-\phi_2$ to be of the order of $\phi\sim 1/M\ll 1$. Then one rescales $\phi\equiv\tilde{\phi}/M$, substitutes $\tan\phi\sim\tilde{\phi}/M$ in the ''channel factor'' Eq. (3) and performs the limit $M\rightarrow\infty$ explicitly. For $\omega=0$ the resulting expression turns out to be *identical* to the parametric correlation function of eigenvalues of large random matrices in the crossover regime derived for the first time by N. Taniguchi *et al.* [25]. Taking into account that $\phi_1-\phi_2\sim 1/M$ results also in $\theta_1-\theta_2\sim 1/M$, we conclude that the statistics of scattering phase shifts in the large M limit is just the same as that of energy levels of *closed* chaotic systems. The latter conclusion is in agreement with the available numerical results obtained for a realistic model of chaotic systems with $M=23$ in [9]. It is also interesting, that the only modification required for $\omega\neq 0$ is to replace $\tilde{\phi}\rightarrow\tilde{\phi}+\omega$.

Considering the time delay characteristics one should take into account that the width of the time delay distribution is of lower order in M as compared with the mean value $\langle \tau_W \rangle$ when resonances are overlapping: $MT \gg 1$ [19]. To extract the time delay correlations in the corresponding limit requires a calculation similar to that done in the paper by Pluhar *et al.* [25]. The resulting expression turns out to be quite a transparent one and is given by

$$K^{\tau}_{\omega,x,y_1,y_2} = \frac{1}{2}\left(\frac{\Gamma_-^2 - \omega^2}{[\Gamma_-^2 + \omega^2]^2} + \frac{\Gamma_+^2 - \omega^2}{[\Gamma_+^2 + \omega^2]^2} \right) \qquad (8)$$

provided that $\Gamma_\pm \equiv MT/2 + x^2 + (y_1 \pm y_2)^2 \gg 1$. Actually, this formula is nothing else but the semiclassical expression for the time delay correlator. It can be obtained from the Gutzwiller trace formula in diagonal approximation, with the quantity $MT/2$ being replaced by the decay rate out of the chaotic region, see [14,18].

The last point to be mentioned is related to the issue of fluctuations of low-frequency admittance as defined in [13]. We noted above that the time delay fluctuates weakly in the many-channel limit $M \to \infty$. Using this fact and the relation between the time delay and the low-frequency admittance $G^I(\omega)$ presented in [13] (see, however, [26]) one finds that the parametric correlator $\langle G^I(0,y_1) G^I(x,y_2) \rangle / \langle G^I \rangle^2 - 1$ of the admittance in the limit $M \gg 1$ is given by

$$\frac{1}{2}\left(\frac{1}{\Gamma_-^2} + \frac{1}{\Gamma_+^2} \right)\left(1 + \frac{Me^2}{C_e \Delta} \langle \tau_W \rangle \right)^{-2}, \qquad (9)$$

where C_e denotes the so called ''geometric capacitance'' [13] and e stands for the electron charge. In the limiting cases of unbroken $(y_1 = y_2 = 0)$ and completely broken $(y_1 = y_2 \to \infty)$ TRI this expression coincides with that found recently by another method by Brouwer and Büttiker [13].

We are very grateful to V.V. Sokolov for instructive discussions and to Professor Hong Guo for kindly providing us with the numerical data used to extract the time delay distribution depicted in Fig. 1. Financial support from SFB 237 der Deutschen Forschungsgemeinschaft, as well as from the program ''Quantum Chaos'' (Grant No. INTAS-94-2058) is acknowledged with thanks.

[1] E. Wigner, Phys. Rev. **98**, 145 (1955); F.T. Smith, *ibid.* **118**, 349 (1960); **119**, 2098 (1960).

[2] E. Akkermans *et al.*, Phys. Rev. Lett. **66**, 76 (1991).

[3] B.L. Altshuler and B.D. Simon, in *Mesoscopic Quantum Physics*, Proceedings of the Les Houches Summer School, Session LXI, edited by E. Akkermans *et al.* (Elsevier, Amsterdam, 1995), p. 5.

[4] U. Smilansky, in *Chaos and Quantum Physics*, Proceedings of the Les Houches Summer School, Session LII, edited by M.J. Giannoni *et al.* (North-Holland, Amsterdam, 1991), p. 372.

[5] P. Gaspard, in *Quantum Chaos*, Proceedings of Enrico Fermi Summer School, edited by G. Casati *et al.* (North-Holland, Amsterdam, 1991), p. 307.

[6] E. Doron, U. Smilansky, and A. Frenkel, Phys. Rev. Lett. **65**, 3072 (1990); and in [5], p. 399.

[7] A. Shushin and D.W. Wardlaw, J. Phys. A **25**, 1503 (1992).

[8] R.A. Jalabert, J.-L. Pichard, J. Phys. (France) I **5**, 287 (1995); E. Mucciolo, R.A. Jalabert, and J.-L. Pichard, Report No. cond-mat/9703178 (unpublished).

[9] B. Dietz, M. Lombardi, and T.H. Seligman, Phys. Lett. A **215**, 181 (1996).

[10] Y. Wang, N. Zhu, J. Wang, and H. Guo, Phys. Rev. B **53**, 16 408 (1996).

[11] P.W. Anderson and P.A. Lee, Prog. Theor. Phys. Suppl. **69**, 212 (1980).

[12] E. Buks *et al.*, Phys. Rev. Lett. **77**, 4664 (1996).

[13] V. Gopar, P. Mello, and M. Büttiker, Phys. Rev. Lett. **77**, 3005 (1996); P. Brouwer and M. Büttiker, Report No. cond-mat/9610144.

[14] B. Eckhardt, CHAOS **3**, 613 (1993).

[15] J.J.M. Verbaarschot, H.A. Weidenmüller, and M.R. Zirnbauer, Phys. Rep. **129**, 367 (1985).

[16] K.B. Efetov, Adv. Phys. **32**, 53 (1983).

[17] Y.V. Fyodorov and H.-J. Sommers, Phys. Rev. Lett. **76**, 4709 (1996); Zh. Eksp. Teor. Fiz. Pis'ma Red. **63**, 970 (1996) [JETP Lett. **63**, 1026 (1996)].

[18] Y.V. Fyodorov and H.-J. Sommers, J. Math. Phys. **38**, 1918 (1997).

[19] N. Lehmann, D.V. Savin, V.V. Sokolov, and H.-J. Sommers, Physica D **86**, 572 (1995).

[20] H.L. Harney, F.M. Dittes, and A. Müller, Ann. Phys. (N.Y.) **220**, 159 (1992).

[21] P. Seba, K. Zyczkowski, and J. Zakrzewski, Phys. Rev. E **54**, 2438 (1996).

[22] A.M.S. Macedo, Phys. Rev. E **50**, R659 (1994).

[23] A. Pandey and M.L. Mehta, Commun. Math. Phys. **16**, 2655 (1983); J.B. French *et al.*, Ann. Phys. (N.Y.) **181**, 198 (1988).

[24] N. Dupius and G. Montambaux, Phys. Rev. B **43**, 14 390 (1991); K. Zyczkowski and G. Lenz, Z. Phys. B **82**, 299 (1991); H.-J. Sommers and S. Iida, Phys. Rev. E **49**, 2513 (1994); V.I. Fal'ko and K.B. Efetov, Phys. Rev. B **50**, 11 267 (1994); Phys. Rev. Lett. **77**, 912 (1996); O. Bohigas *et al.*, Nonlinearity **8**, 203 (1995); Y. Alhassid *et al.*, Report No. cond-mat/9609115; S.A. van Langen *et al.*, Phys. Rev. E **55**, 1 (1997).

[25] A. Altland *et al.*, Ann. Phys. (N.Y.) **219**, 148 (1992); N. Taniguchi *et al.*, Europhys. Lett. **27**, 335 (1994); A. Altland *et al.*, J. Phys. A **26**, 3545 (1993); Z. Pluhar *et al.*, Ann. Phys. (N.Y.) **243**, 1 (1995).

[26] The effects of charge quantization not taken into account in [13] may lead to modification of the results, see I.L. Aleiner and L.I. Glazman, cond-mat/9612138.